An Attitude Stability Accommodating Guidance Technique for a Launch Vehicle

Jae Weon Choi* and Joongsun Yoon*

(*Received March 30, 1998*)

Conventional closed-loop guidance commands are usually generated from a simplified pointmass model. In both real situations and computer simulations, however, the generated guidance commands are applied to a full rigid body model, which can cause attitude instabilities in a vehicle. In this paper, the influence of guidance commands, generated from a point mass model, on the full rigid body model is analyzed quantitatively, and an attitude stabilizing strategy is proposed. For this purpose, an attitude stabilizing condition for the rigid body model is derived using the Lyapunov Stability Theorem. The derived stabilizing condition can be used to stabilize the attitude of a launch vehicle by applying the corresponding feedforward guidance commands when the commands generated from a point mass model destabilize or undeterminate the attitude stability of a launch vehicle. Computer simulations have been performed to verify our conjecture for the second stage of the Japanese N-I launch vehicle.

Key Words: Attitude Stabilizing Condition, Lyapunov Stability Theorem, N-I Launch Vehicle.

1. Introduction

Conventional guidance commands for aerospace vehicles, which may be determined using optimization technigues such as the calculus of variations (Gelfand and Fomin; Kirk, 1970) or parametric optimization (Ceballos, 1986), are generated from a simplified point mass model, *viz*. 3-degree-of-freedom (3DOF) model. In both real situations and computer simulations, however, the guidance commands are applied to a full rigid body 6DOF model that considers the rotational dynamics. The distinction between a 3DOF and a 6DOF simulation is that the 3DOF simulation integrates only the translational equations of motion and assumes instantaneous rotational response to steering commands. Vehicle inertial resistance to torques is not modeled, and steering through instantaneous steady-state angular reorientation is affected. Because of the additional differential equations in the 6DOF model, attitude instability of the vehicle can occur.

In Luke, (1993), guidance system approximation and trajectory optimization were discussed, and the author showed that the resulting impacts on a 3DOF trajectory in terms of vehicle attitude and angle-of-attack time histories. Otsubo *et. al.* (1983), Lee, Choi, and Lee (1992), Choi (1998), also pointed out that guidance commands which are generated in the sense of minimum fuel in a simplified point mass model may cause attitude instability of a launch vehicle. This implies that the closed-loop attitude stability of the launch vehicle may not be guaranteed because of the additional rotational dynamic equations. Since this may result in poor control performance characteristics, it is required to analyze the causes and determine solution for this problem.

In this paper, the influence of guidance commands that are generated from a point mass model on the full rigid body model is analyzed

* Assistant Professor, School of Mechanical Engineering and Research Institute of Mechanical Technology, San 30, Jangjeon-Dong, Keumjung-Ku, Pusan 609-735, Korea.

quantitatively, and an attitude stabilizing strategy is proposed. For this purpose, an attitude stabilizing condition for the rigid body model is derived using the Lyapunov Stability Theorem. The derived stabilizing condition can be used to stabilize the attitude of a launch vehicle by applying the corresponding feedforward guidance commands when the commands generated from a point mass model destabilize or *undeterminate* the attitude stability of a launch vehicle. To verify our conjecture, simulations have been carried out for the second stage of the Japanese N-I launch vehicle.

The results of this paper may be used to reduce unnecessary fuel consumption for stabilizing the unstable attitude of a launch vehicle by excluding the guidance commands which violate the proposed attitude stability condition. In addition, they provide a theoretical basis that confirms that the generated guidance commands satisfying the proposed attitude stability condition are applicable to the real world without loss of generality.

2. Influence Effect of the Guidance Commands on Attitude of a Launch Vehicle

In this section, we first analyze quantitatively the effect that guidance commands generated from a point mass model have on a full rigid body model, *viz*. translational and rotational equations of motion. Then, we examine closely the mutual relation between the effect and the attitude instability of a launch vehicle. Finally, a useful strategy for solving the attitude instability problem is proposed.

2.1 Qumtification

In this paper, we assume that the aerodynamic forces and moments are negligible, and that the thrust force and mass flow rate of the launch vehicle are constant. Then, the guidance equations that generate the desired guidance commands in the body coordinate frame (X_B, Y_B, Z_B) are represented as follows:

$$\dot{U}=\frac{T_g}{m}\cos\beta_y\cos\beta_z+G_{Bx}$$

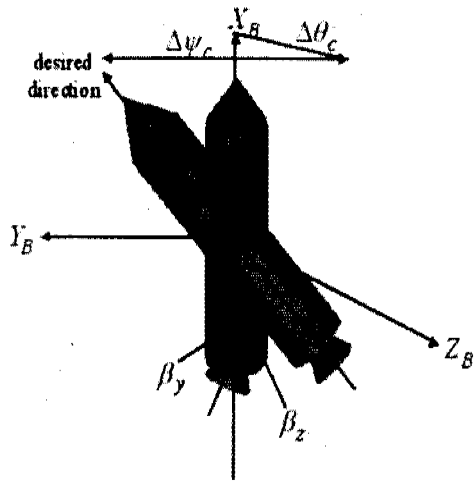


Fig. 1 Rigid motion model of launch vehicle.

$$\dot{V}=-\frac{T_g}{m}\sin\beta_z+G_{By} \tag{1}$$

$$\dot{W}=\frac{T_g}{m}\sin\beta_y\cos\beta_z+G_{Bz}$$

where, U=forward velocity of the launch vehicle in the body frame (X_B-axis), V=side velocity of the launch vehicle in the body frame (Y_B-axis), W=downward velocity of the launch vehicle in the body frame (Z_B-axis), m=present mass of the launch vehicle, T_g=thrust force, $G_B=[G_{Bx}, G_{By}, G_{Bz}]$=gravitational acceleration vector in the body frame, β_y=engine tilt angle (pitch) or equivalently pitch guidance command, and β_z= engine tilt angle (yaw) or equivalently, the yaw guidance command. The pitch and yaw guidance commands β_y, β_z are shown in Fig. 1, which depicts a rigid motion model of the launch vehicle. Note that the desired guidance commands are calculated using the above point mass model (translational equations of motion); that is, the rotational dynamics are not considered in the calculation of the guidance commands.

The guidance commands which are generated using the point mass model (*Eq*. (1)) in the sense of minimum fuel or any other performance index (Lee, 1995), are input to the following full rigid body model. It is clear from the equation of motion that, the position, velocity, and attitude of a rigid launch vehicle can be affected by the commands.

Translational Equations of Motion:

$$\dot{U}=\frac{T_g}{m}\cos\beta_y\cos\beta_z+G_{Bx}+RV-QW$$
$$\dot{V}=-\frac{T_g}{m}\sin\beta_z+G_{By}+PW-RU$$
$$\dot{W}=\frac{T_g}{m}\sin\beta_y\cos\beta_z+G_{Bz}+QU-PV$$

Rotational Equations of Motion:

$$\begin{aligned}\dot{P}&=\frac{M_{gx}}{I_x}+\frac{I_y-I_z}{I_x}RQ\\ \dot{Q}&=\frac{M_{gy}}{I_y}+\frac{I_z-I_x}{I_y}PR\\ \dot{R}&=\frac{M_{gz}}{I_z}+\frac{I_x-I_y}{I_z}PQ\end{aligned} \tag{3}$$

where, P, Q, R are roll, pitch, yaw rates in the body frame, respectively, M_g=rotational moment by thrust force, and I=inertial moment of the launch vehicle.

Now we rewrite the rotational equations of motion (*Eq*. (3)) by substituting appropriate quantities for the rotational moment M_g in order to investigate the effect of the guidance commands on the attitude of the launch vehicle quantitatively (refer to Fig. 2.):

$$\begin{aligned}\dot{P}&=\frac{I_y-I_z}{I_x}RQ\\ \dot{Q}&=\frac{I_z-I_x}{I_y}PR+\frac{L_gT_g}{I_y}\sin\beta_y\cos\beta_z\\ \dot{R}&=\frac{I_x-I_y}{I_z}PQ+\frac{L_gT_g}{I_z}\sin\beta_z\end{aligned}$$

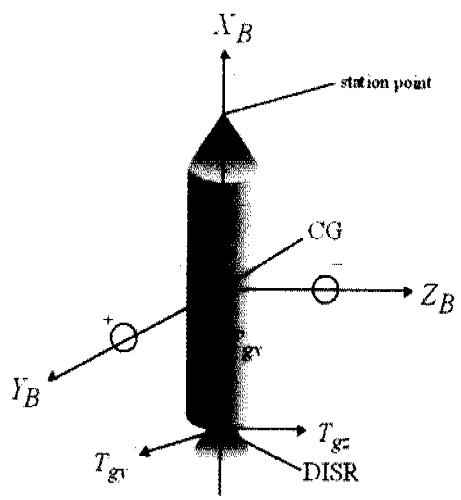


Fig. 2 Rotational moment by thrust force.

where, L_g is the distance from CG (Center of Gravity) to DISR (Center of Thrust Force). In *Eq*. (4), the second terms of the equations may destabilize the attitude of the launch vehicle.

2.2 Attitude stabilizing condition via lyapunov analysis

In this subsection, we utilize the following Lyapunov Stability Theorem to find stable regions for the vehicle attitude.

Theorem 1: (see, e.g., Narendra and Annaswamy, 1989)
The equilibrium state of the nonlinear equation $\dot{x}=f(x, t)$, $f(0, t)=0$ for $\forall t\geq t_0$ is uniformly asymptotically stable in the large if a scalar function $V(x, t)$ with continuous first partial derivatives with respect to x and t exists such that $V(0, t)=0$ and the following conditions are satisfied:

1. $V(x, t)$ is *positive-definite*, that is, there exists a continuous nondecreasing scalar function α such that $\alpha(0)=0$ and $V(x, t)\geq\alpha(\|x\|)>0$ for all t and all $x\neq0$;
2. $V(x, t)$ is *decrescent*, that is, there exists a continuous nondecreasing scalar function β such that $\beta(0)=0$ and $\beta(\|x\|)\geq V(x, t)$ for all t;
3. $\dot{V}(x, t)$ is *negative-definite*, that is,

$$\dot{V}(x, t)=\frac{\partial V}{\partial t}+(\nabla V)^{\mathrm{T}}f(x, t)<-\gamma(\|x\|)<0$$

where γ is a continuous nondecreasing scalar function with $\gamma(0)=0$, ∇V is the gradient of V with respect to x, and the time derivative of V is evaluated along the trajectory of the given differential equation.

4. $V(x, t)$ is *radially unbounded viz.*, $\alpha(\|x\|)\rightarrow\infty$ with $\|x\|\rightarrow\infty$.

In *Eq*. (4), $\dot{P}=0$ because $I_y=I_z$ by symmetry of the launch vehicle, and only the two equations in Q and R remain. Thus, we can choose a Lyapunov function candidate as follows:

$$V=\frac{1}{2}(Q^2+R^2). \tag{5}$$

The time rate of V in the following equation should be negative regardless of the effect of the guidance commands in order that the launch vehicle attitude may be asymptotically stable:

$$\dot{V} = Q\dot{Q} + R\dot{R} = \frac{L_g T_g}{I_y}(\sin\beta_y \cos\beta_z Q + \sin\beta_z R) \quad (6)$$

In this equation, the term $\frac{L_g T_g}{I_y}$ is always positive, and $\cos\beta_z$ is also positive because the guidance commands β_y, β_z are bounded by ± 90 degrees. Thus, the following condition should be satisfied in order that the attitude may be asymptotically stable:

$$Q\sin\beta_y + R\tan\beta_z < 0. \quad (7)$$

The detailed attitude stability conditions according to the conditions of each variable in *Eq*. (7) are the following:

1. If $Q\sin\beta_y > 0$ and $R\tan\beta_z > 0$, then the attitude is undetermined,
2. elseif $Q\sin\beta_y > 0$ and $R\tan\beta_z < 0$, and $|R\tan\beta_z| > |Q\sin\beta_y|$, then the attitude is asymptotically stable,
3. elseif $Q\sin\beta_y < 0$ and $R\tan\beta_z > 0$, and $|R\tan\beta_z| < |Q\sin\beta_y|$, then the attitude is asymptotically stable,
4. elseif $Q\sin\beta_y < 0$ and $R\tan\beta_z < 0$, then the attitude is always asymptotically stable,
5. elseif $Q\sin\beta_y = 0$ and the attitude is
   - $R\tan\beta_z > 0$, then the attitude is undetermined,
   - $R\tan\beta_z = 0$, then the attitude is stable,
   - $R\tan\beta_z < 0$, then the attitude is then always asymptotically stable,
6. elseif $R\tan\beta_z = 0$ and
   - $Q\sin\beta_y > 0$, then the attitude is undetermined,
   - $Q\sin\beta_y > 0$, then the attitude is always asymptotically stable.

Note that the undetermined cases in the above conditions can be interpreted as either (asymptotically) stable or unstable, because the Lyapunov Stability Theorem provides only a sufficient condition.

2.3 Elimination of the guidance commands causing attitude instability

In this subsection, it is shown via simulations that the generated guidance commands which correspond to the *undetermined* cases in the conditions described in the previous subsection may cause attitude instability in a launch vehicle. Because of the inherent nature of the launch vehicle, the attitude of the vehicle changes rapidly when certain events such as stage separation or engine cutoff occur. In these cases, the guidance commands could cause the attitude of the vehicle to become unstable. In this paper, a strategy that overcome this problem is proposed. The strategy is based on the fact that the effect of the guidance commands which destabilize or *undeterminate* the attitude of the vehicle is known value as follows:

- Check stability using the condition described in the previous section when certain events such as stage separation or engine cutoff occur.
- If the result of the check is *undetermined*, then the corresponding guidance command is eliminated. Thus, through these processes, the attitude of the vehicle becomes stable. From the viewpoint of the guidance system, the eliminated quantity can be considered by a feedforward guidance command.

3. Simulation Results

In this section, it is verified via simulations that closed-loop guidance commands may cause attitude instability of the vehicle, and also it is confirmed that the vehicle attitude can be stabilized by the proposed strategy described in the previous section.

For the simulations, Tanegashima Island of Japan is considered as a launch site for the Japanese launch vehicle N-I (Lee *et al.* 1988). The open-loop pre-program guidance scheme is used for guiding the first stage of the vehicle, while for the second stage, a closed-loop explicit guidance scheme (Hough, 1988; Choi, 1998) is used. The launch vehicle flies according to the event sequence described in Table 1.

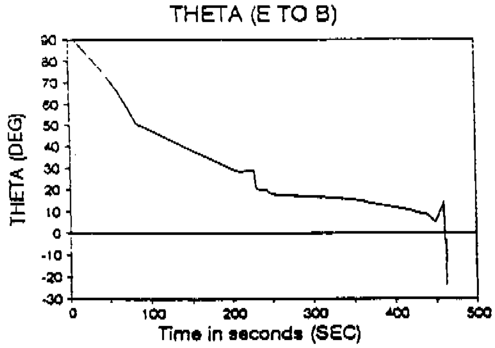
Initially, simulations have been performed for verifying the fact that some guidance commands which generated from a simplified point mass model of a launch vehicle may cause attitude instability in the vehicle. The results are shown in Fig. 3. Then, it is confirmed that the attitude of

Table 1 Events sequence.

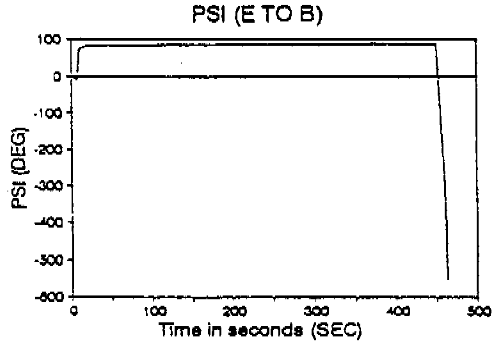
Time (sec)	Events
0.00	First stage main engine, SOB, vernier engine ignition
7.33	Pitch program (No. 1) start
38.19	SOB cut off
40.00	Pitch program (No. 1) finish
40.33	Pitch program (No. 2) start
60.00	Pitch program (No. 2) finish
60.33	Pitch program (No. 3) start
80.00	SOB separation
85.00	Pitch program (No. 3) finish
85.33	Pitch program (No. 4) start
208.00	Pitch program (No. 4) finish
218.513	First stage main engine cut off (MECO)
219.513	Remaining thrust for the first stage main engine cut off
224.513	Vernier engine cut off (VECO)
226.013	First stage separation
228.013	Second stage main engine ignition (roll-gas jet: controllable state) Guidance program for the second stage start (explicit guidance scheme)
242.513	Jettison of the fairing cover
461.970	Second stage main engine cut off (SECO)

the vehicle can be stabilized by eliminating the *undetermined* guidance command via feedforwarding which is based on a strategy proposed in the previous section.

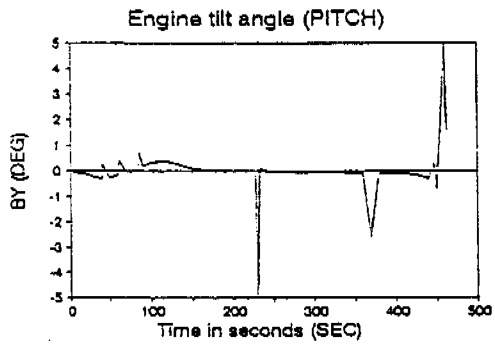
The time history of the pitch attitude angle of the launch vehicle N-I is depicted in Fig. 3 (a). In the figure, we find that a rapid change occurs when the second main engine is cut off (461. 970 sec). This phenomenon leads to poor control characteristics, and causes failure in attitude control during inertial flight of the second stage after 463 sec. Hence, it is required to slow down the rate of change of the vehicle attitude by stabilizing the attitude. In addition, the slight attitude variations in the vicinity of 230 sec can be understood due to the second stage separation and the jettisoning of the fairing cover, and can be neglected because the variations are small. Figure 3 (b) shows the time history of the yaw attitude



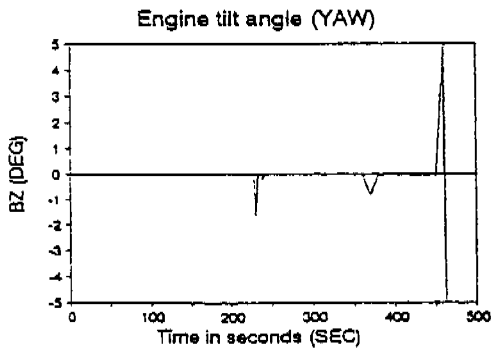
(a) Pitch attitude angle



(b) Yaw attitude angle

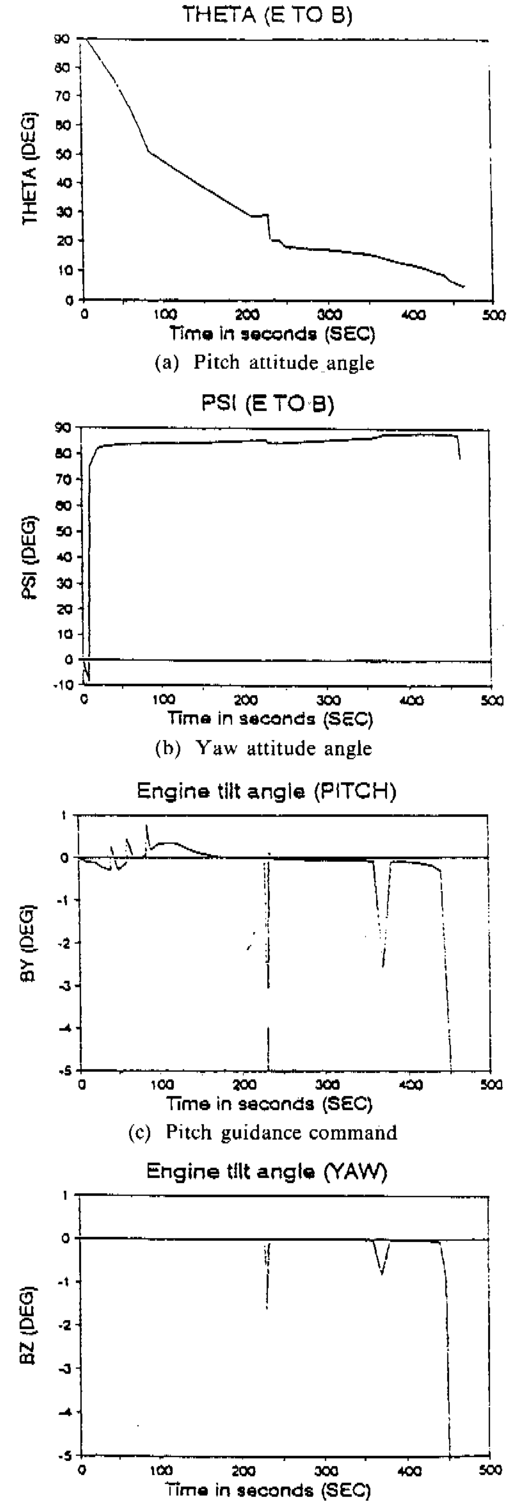


(c) Pitch guidance command



(d) Yaw guidance command

Fig. 3 Unstable vehicle attitude caused by the *undetermined* guidance commands.



(a) Pitch attitude angle

(b) Yaw attitude angle

(c) Pitch guidance command

(d) Yaw guidance command

Fig. 4 Attitude stabilization by eliminating the *undetermined* guidance commands.

angle of the vehicle. In this Figure, we can also find severe attitude instability in the vicinity of 460 sec. Figures (c) and (d) of Fig. 3 denote the time histories of the the pitch and yaw guidance commands, respectively.

Figure 4 (a) and (b) depict the time histories of the pitch and yaw attitude angle when eliminating the *undetermined* guidance commands from the rotational equations of motion after checking the stability condition described in the previous section. In the figures, rapid attitude changes are removed. Fig. 4(a) and (d) show the time histories of the guidance commands when removing the commands that cause attitude instability of the vehicle. In Fig. 3 (c), (d) and Fig. 4(c), (d), we find that the used input quantities of the two cases are almost the same. This means that the vehicle attitude can be stabilized without additional fuel consumption by feedforward guidance commands, that is, by the compensated guidance commands from the guidance system viewpoint.

4. Concluding Remarks

In this paper, the influence of guidance commands generated from a point mass model on a full rigid body model is analyzed quantitatively, and verified by simulations for the second stage of the Japanese N-1 launch vehicle. An attitude stabilizing condition for the rigid body model is derived using the Lyapunov Stability Theorem. The derived stabilizing condition can be used to stabilize the attitude of a launch vehicle without additional fuel consumption by applying the corresponding feedforward guidance commands when the commands generated from the point mass model destabilize the attitude of the launch vehicle. The results of this paper provide a theoretical basis that confirms that the generated guidance commands satisfying the attitude stability condition are applicable to the real world without loss of generality.

Reference

Ceballos, D. C., 1986, "Compensating Structure and Parameter Optimization for Attitude

Control of a Flexible Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 2, pp. 248～249.

Choi, J. W., 1998, "Performance Analysis of an Explicit Guidance Scheme for a Launch Vehicle," *Journal of the Korean Society of Precision Engineering*, Vol. 15, No. 6, pp. 97～106, (in Korean).

Gelfand, I. M., and Fomin, S. V., *Calculus of Variations*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Hough, M. E., 1988, "Explicit Guidance Along an Optimal Space Curve," *AIAA paper 88-4297-CP*, pp. 620～626.

Kirk, D. E., 1970, *Optimal Control Theory: An Introduction*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Lee, B., 1995, "Chaos Maximizing Optimal Control," *KSME International Journal*, Vol. 9, No. 4, pp. 397～409.

Lee, J. G., Choi, J. W., Rho, O.-H., and Han, M. K., 1988, *Basic Research on Spacecraft Guidance and Control System*, Final Report, Ministry of Science and Technology, Korea, (in Korean).

Lee, S. H., Choi, J. W., and Lee, J. G., 1992, "Optimal Reference Trajectory Analysis of a Launch Vehicle for Scientific Missions," *Journal of the Korean Society for Aeronautical and Space Sciences*, Vol. 20, No. 1, pp. 27～38, (in Korean).

Luke, R. A., 1993, "Rotational Motion and Guidance System Approximations in Optimizable Operational Launch Vehicle Simulation," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 3, pp. 477～483.

Narendra, K. S., and Annaswamy, A. M., 1989, *Stable Adaptive Systems*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Otsubo, K., Oguchi, M., Nitta, K., and Mori, H., 1983, *Strapdown Inertial Guidance System: Onboard Software and Its Evaluation*, Technical Report of National Aerospace Laboratory, Japan, TR-775, (in Japanese).